Analysis of VR stepper motor dynamics considering 3D FE model with core lamination and effect of minimum energy control on steel loss

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Abstract — **This research presents a method for the simulation of the variable reluctance stepper motor dynamics taking motion, eddy currents and magnetic non-linearity into account. The proposed 3D model is used to obtain the optimal minimum energy control law that minimizes the energy injected by the controller. In the paper the open – loop control problem is analyzed for the square-wave voltage excitation. Next to demonstrate the effectiveness of the proposed optimal minimum energy control method is applied. In both cases, the examination of the four phased variable reluctance stepper motor dynamics and the steel loss in the core is presented and compared.**

I. INTRODUCTION

Stepper motors are the one of the most popular motor types and are the products of the multidisciplinary areas including electrical, mechanical or control engineering.

In literature, there are many papers related to the design, modeling or control of the switching reluctance motors [2,3,5-7]. From the energy consumption point of view, very interesting are papers deal with the optimization of the excitation for the FE models of the stepper machines [5-7]. This paper presents the optimal control based method to determine the optimal exciting voltage considering the energy delivered to the motor. The optimal waveform is necessary to reduce the energy injected by the control inputs of the device.

As a new contribution the optimal control sequence is proposed for the non-linear, eddy current FE model of the stepper motor. The control problem is designed and analyzed employing the feedback gain strategy for the infinite time horizon problem.

II. THE MODEL

The configuration of the variable reluctance stepper motor and the sequence of the voltage waveforms is shown in Fig. 1. The governing equations of the electromagnetic field consider the eddy current problem in the stator and the rotor core:

$$\nabla\times\left(\frac{1}{\mu}\nabla\times\mathbf{A}\right)+\left(\sigma\frac{\partial\mathbf{A}}{\partial t}+\sigma\nabla V-\sigma\mathbf{v}\times\left(\nabla\times\mathbf{A}\right)\right)-\mathbf{j}=0 \quad (1)$$

$$-\nabla\cdot\left(\sigma\frac{\partial\mathbf{A}}{\partial t}+\sigma\nabla V-\sigma\mathbf{v}\times\left(\nabla\times\mathbf{A}\right)\right)=0 \quad (2)$$

where: $\mathbf{A}$ denotes magnetic vector potential V - electric scalar potential, μ - permeability, σ - conductivity, $\mathbf{v}$ - rotor speed

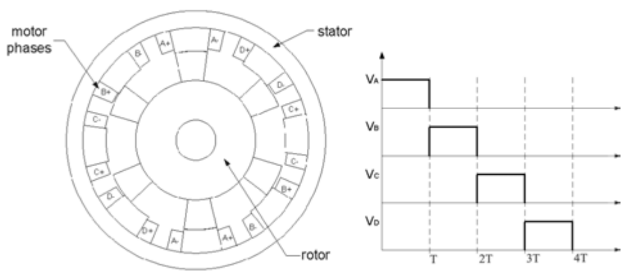

Fig. 1. The motor configuration and voltage waveforms

and $\mathbf{j}$ is a winding current density. In this model the non-linearity of the rotor and the stator materials is considered. The linearization is performed employing the Newton – Raphson procedure [1].

The stator phase circuit equation for the described stepper motor is

$$\frac{d}{dt}\oint_{l_s}\mathbf{Adl}=u_s-Ri_s\,, \quad (3)$$

where s={1,..,4} denotes the phase number, R is a winding resistance, i is a phase current and u is a supply voltage.

The rotor displacement is evaluated by solution of the mechanical motion equation:

$$J\frac{d^2\Theta}{dt^2}+b\frac{d\Theta}{dt}=T\,, \quad (4)$$

where Θ is the rotor displacement, J - rotor inertia, b - damping, T - electromagnetic and load torque.

The solution of the equation (1)-(2) is obtained by means of the finite element method using 27-node, first order cylindrical elements. The boundary value problem (1)-(2) is coupled to the electric circuit equations (3) and the global field – circuit coupled system of equation is solved [1].

The discrete rotor speed and displacement are determined using the backward Euler's approximation for the equation (4). The motion in the electromagnetic field is realized using the fixed grid technique. The moving body displacement is updating (if necessary) at each iteration step.

The global torque is calculated using the Maxwell stress method. The force is evaluated along a surface in the airgap around the rotor employing the eggshell approach.

III. THE OPTIMAL CONTROLLER

The aim of the authors is to obtain the optimal control for the FE motor model that minimizes the energy injected by the control inputs. The control problem consists of finding a voltage control vector $\mathbf{U} = [u_1 \quad u_2 \quad u_3 \quad u_4]^T$ applied on the motor windings. The problem is solved via the linear quadratic programming method LQP [4].

Employing this model, the winding currents may be controlled by the voltage functions as an admissible controls $\mathbf{U} \in D_u$. Then voltage control should minimize the objective function:

$$J(\mathbf{U}) = \frac{1}{2}\int_0^\infty \mathbf{U}^T \mathbf{P}\mathbf{U} dt \,, \tag{5}$$

where $\mathbf{U}^T\mathbf{PU}$ is a function of the power injected by the controller. Basing on the LQP, the voltage control function may be determined minimizing $J(\mathbf{U})$. Solving the problem, the control law may be defined as the function of the current $\mathbf{I}$ and reference current vector $\mathbf{I}_{ref}$.

$$\mathbf{U}^* = -\mathbf{P}^{-1}\mathbf{\Gamma}\left(\mathbf{I} - \mathbf{N}\mathbf{I}_{ref}\right) \tag{6}$$

The matrix $\mathbf{\Gamma}$ is determined from the non-linear Riccati equation [4]. The matrix $\mathbf{N}$ is related to $\mathbf{I}_{ref}$ and $\mathbf{\Gamma}^{-1}$, $\mathbf{P}$, $\mathbf{R}$.

Figure 2 shows the procedure to solve the electromechanical system of the motor closed to the loop of the optimal control.

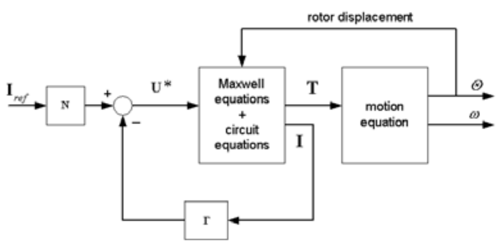

Fig. 2. Closed loop procedure for solving the optimal control problem

In this process the voltage equation is modified by considering the current in the motor windings and is coupled with the Maxwell equation to obtain the time-stepping finite element equation for the analysis of the electromagnetic field. Then the torque $\mathbf{T}$ is computed and the motion equation is solved for a new position of the rotor. The new position is considered for the recalculation of the field in the next step.

IV. NUMERICAL EXPERIMENT

The control technique is applied to the analysis of the four-phase variable reluctance motor. There is examined the stepping motor as the control plant using the 3D approach (Fig. 3), in which the non-linearity and eddy current problem are considered.

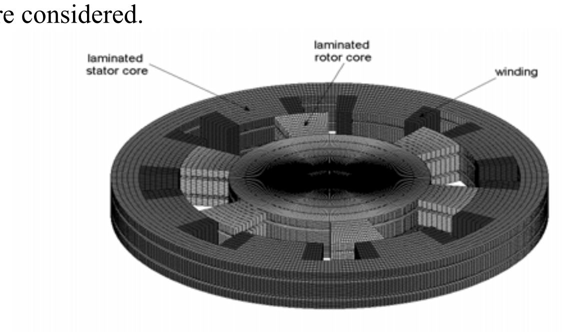

Fig. 3. The mesh of analyzed motor

Firstly, the open – loop control problem is analyzed for the square-wave voltage excitation with amplitude u=30 V. Next, to demonstrate the proposed closed – loop control method, the voltage profile is computed for the reference current i_{ref} =2 A. Fig. 4 shows the step response comparison and Fig. 5 presents the power losses in the laminated core for both cases.

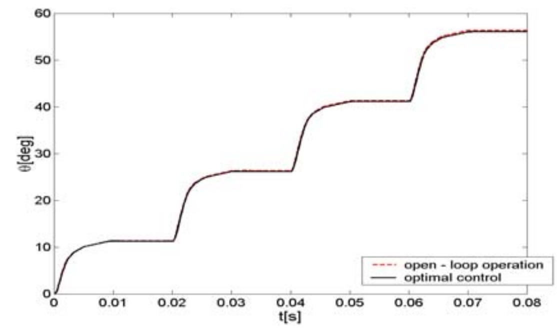

Fig. 4. Comparison of step response

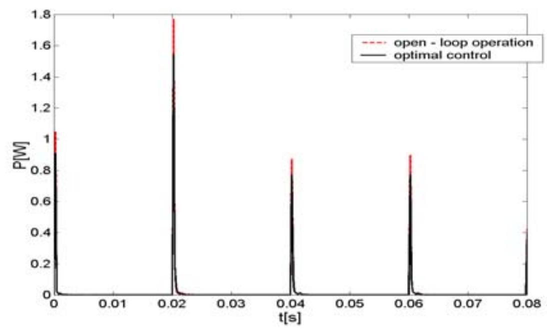

Fig. 5. Comparison of power loss

In the full paper also the voltage control, the current response will be presented and compared with measurement.

V. CONCLUSION

The presented optimal control strategy, may enable to save the part of the energy provided by the controller, the energy lost on the heat in the electric circuit and the power loss in the motor core.

VI. REFERENCES

[1] W. Fu and S. Ho, "Enhanced nonlinear algorithm for the transient analysis of magnetic field and electric circuit coupled problems" *IEEE Trans. Magn*., Vol. 45, No. 2, pp. 701–706, 2009

[2] R. Krishnan, "Switched Reluctance Motor Drives. Modeling, Simulation, Analysis, Design, and Applications", ser. I. E.. Boca Raton, FL: CRC Press, 2001.

[3] K. Vijayakumar, R. Karthikeyan, S. Paramasivam, R. Arumugam and K. Srinivas, "Switched Reluctance Motor Modeling, Design, Simulation, and Analysis: A Comprehensive Review", *IEEE Trans. Mag*., Vol. 44, No. 12, pp. 4605-4617, 2008

[4] W. Haddad and V. Chellaboina, "Nonlinear Dynamical Systems and Control: A Lyapunov-Based Approach", Princeton University Press, 2008

[5] S. Bolognani and M. Zigliotto, "Fuzzy logic control of a switched reluctance motor drive," *IEEE Trans. Ind. Appl*., vol. 32, no. 5, pp. 1063–1068, 1996

[6] S. Mir, M. Islam, T. Sebastian and I. Husain, "Fault-tolerant switched reluctance motor drive using adaptive fuzzy logic controller", *IEEE Trans. Power Electron*., vol. 19, no. 2, pp. 289–295, 2004

[7] S. Paramasivam and R. Arumugam, "Real time hybrid controller implementation for switched reluctance motor drive," *Amer. J. Appl. Sci.*, vol. 1, no. 4, pp. 284–294, 2004